

C. U. SHAH UNIVERSITY

Summer Examination-2022

Subject Name : Problem Solving-II

Subject Code : 5SC03PRS1

Branch: M.Sc. (Mathematics)

Semester: 3

Date: 27/04/2022

Time: 02:30 To 05:30

Marks: 70

Instructions:

- (1) Use of Programmable calculator and any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

SECTION – I

- Q-1 Attempt the Following questions. [07]**
- a. Find $\Delta^2 x^3$ at $x = 0$. (02)
 - b. Is a group of order 10 simple? Verify. (02)
 - c. Define Monoid. Give two examples of Monoid which are not Groups. (02)
 - d. True or False: $\mathbf{Z} \times \mathbf{Z}$ is a cyclic group. (01)
- Q-2 Attempt all questions [14]**
- a. Let G be a finite abelian group of order n . Show that the map $x \rightarrow x^m$ is an automorphism if $\gcd(m, n) = 1$. (06)
 - b. Let G be a group, H and K be subgroups of G , if $(o(H), o(K)) = 1$ then what we can say about $o(H \cap K)$? (04)
 - c. Use Lagrange's Inverse Interpolation Formula to find x when $f(x) = 14$ given $f(0) = 16.35$, $f(5) = 14.88$, $f(10) = 13.59$ and $f(15) = 12.46$ (04)
- OR**
- Q-2**
- a. Let $H = \langle a \rangle$ and $K = \langle b \rangle$ be two cyclic groups of order m and n respectively such that $(m, n) = 1$ then show that $H \times K$ is cyclic group generated by mn . (06)
 - b. For which values of n , the polynomial $P(x) = x^3 - nx + 2$ is reducible over \mathbf{Q} . (04)
 - c. Find $\Delta(e^{ax} \log bx)$. (04)
- Q-3 Attempt all questions. [14]**
- a. Find the integral surface of following Partial Differential Equation $(y - z)Z_x + (z - x)Z_y = (x - y)$ given $z = 0$ on $xy = 1$. (05)
 - b. Find Particular Integral of $(D^2 - DD' + 2D' - 1)z = x^2 y^2$. (05)
 - c. Using Euler's Modified method find $y(0.3)$ given $y' = 1 - y, y(0) = 0$. Take $h = 0.1$ (04)



OR

Q-3

- a. Evaluate $f(8)$ using Newton's Divided difference formula from the following table: (06)

X	4	5	7	10	11	12
Y	48	100	294	900	1210	2028

- b. Solve the following linear equation by Lagrange's method: (06)

$$y^2p - xyq = x(z - 2y).$$

- c. Let G be a non-abelian group of order p^3 where p is prime then find $o(Z(G))$. (02)

SECTION – II

Q-4

Attempt the Following questions. [07]

- a. Find the number of cosets of $H = (4\mathbf{Z}, +)$ in $G = (\mathbf{Z}, +)$. (02)
b. Classify the following Partial Differential Equation : $x^2r - 2s + t = 0$. (02)
c. Find order of any four elements in $U(20)$. (02)
d. Solve: $\Delta \sin x$. (01)

Q-5

Attempt all questions [14]

- a. Solve $\frac{dy}{dx} = xy + y^2$ given $y(0) = 1$. Find $y(0.1)$ using Runge Kutta's method (take $h = 0.1$). (06)
b. Check whether the following polynomials are irreducible over Q or not. (04)
i) $x^6 + x^3 + 1$
ii) $x^3 - 4x + 2$
c. Find isomorphic group to $U(720)$. (04)

OR

Q-5

- a. Let $o(G) = pq$, p and q are prime numbers with $p < q$. If $p \nmid (q - 1)$ i.e. p does not divide $q - 1$, then show that G is cyclic. (07)
b. Solve the system of equations (07)

$$\begin{aligned} 3x + 4y - z &= 8 \\ -2x + y + z &= 3 \\ x + 2y - z &= 2 \end{aligned}$$

Using Gauss Elimination method.

Q-6

Attempt all questions [14]

- a. Solve the following system of linear equations using Gauss Seidel method: (07)

$$28x + 4y - z = 32, x + 3y + 10z = 24, 2x + 17y + 4z = 35.$$

- b. Solve the heat(diffusion) equation $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{1}{k} \frac{\partial \phi}{\partial t}$ by the method of separation of variables and show that the solution is of the form $\phi(x, y, t) = e^{\pm i(nx + my) - (n^2 + m^2)kt}$ where n and m are some constants. (07)

OR

Q-6

- a. Find the missing value in the following data: (06)



X	1	2	3	4	5
Y	2	5	7	---	32

- b.** Find the number of irreducible monic quadratic polynomial in $\mathbf{Z}_p[X]$, where p is prime. **(05)**
- c.** Solve : $pqz = p^2(xq + p^2) + q^2(yp + q^2)$ using Charpit's method. **(03)**

